



95

11  $f(1) = 4$

$f(m) = 2 f(m-1)$  for  $m \geq 2$

$f(2) = 2 f(1) = 8$   
 $f(3) = 2 f(2) = 16$   
 $f(4) = 2 f(3) = 32$   
 $f(5) = 2 f(4) = 64$   
 $f(6) = 2 f(5) = 128$

10

$f(m) = 2 f(m-1)$   
 $f(m) = 2^m f(1)$

$b_m = 4m - 2$

$b_1 = 4 - 2 = 2$   
 $b_2 = 8 - 2 = 6$   
 $b_3 = 12 - 2 = 10$   
 $b_4 = 16 - 2 = 14$   
 $b_5 = 20 - 2 = 18$

10

Base step

base step  $b_1 = 2$

Recursive step:  $b_{m+1} = b_m + 4$

- 2) 12 colors - male  $\leq 3$  size
- female  $\leq 3$  size

The first part is we choose one of the 12 colors  $\Rightarrow 12$  possible  
 The second part

shut for male or shut for female

The number of shut possible for male is  $12 \times 3 = 36$

The number of shut possible for female is  $12 \times 3 = 36$

The total number of shut is  $72$  types

10

5. A multiple test contains 10 questions.

If each question is answered there are then  $4^{10}$  possible ways to answer the question.

1048576 ways to answer the question.

3) We consider each high school as a box then there are 50 boxes.

One of these boxes must have at least 10 correct answers from the generalized pigeonhole principle.

$$\lceil \frac{N}{50} \rceil = 10$$

$$9 < \frac{N}{50} \leq 10$$

$$450 < N \leq 500$$

$$\boxed{N = 451}$$

Then the minimum number of students is equal to 451 student.

4) 3 men 4 women

The number of members in a committee is equal to 4. Then if we have the same number of men and women then it is a selection of women between 4 and of 2 men between 3.

The first task is to select 2 women from 4 women  $C(4, 2)$   
The second task is to select 2 men from 3 men  $C(3, 2)$

By the product rule the total number of committees that have the same number of men and women

$$\begin{aligned} \text{is } C(4, 2) \cdot C(3, 2) &= \frac{4!}{2!2!} \cdot \frac{3!}{2!} = \frac{4 \times 3}{2} \times \frac{6}{2} \\ &= 6 \times 3 = 18 \text{ committees.} \end{aligned}$$

$$5) \quad x^2 y^2 \quad (2x + 3y)^{10}$$

$$\boxed{n = 10}$$

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j$$

$$\text{so } \boxed{j = 8} \quad \binom{10}{8} (2x)^2 (3y)^8$$

(10)

$$\text{So the coefficient of } x^2 y^8$$

$$= \binom{10}{8} 2^2 3^8$$

$$= 45 \times 4 \times 6561$$

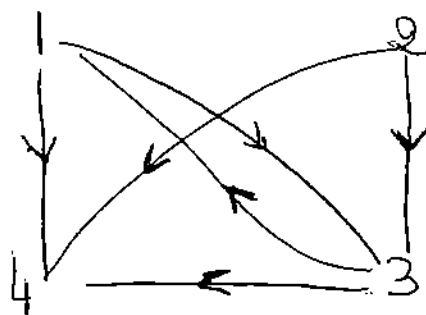
$$= 1180320$$

$$6) \quad R = \{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$$

$$a) \quad M_R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad A = \{1, 2, 3, 4\}$$

b)

(2)



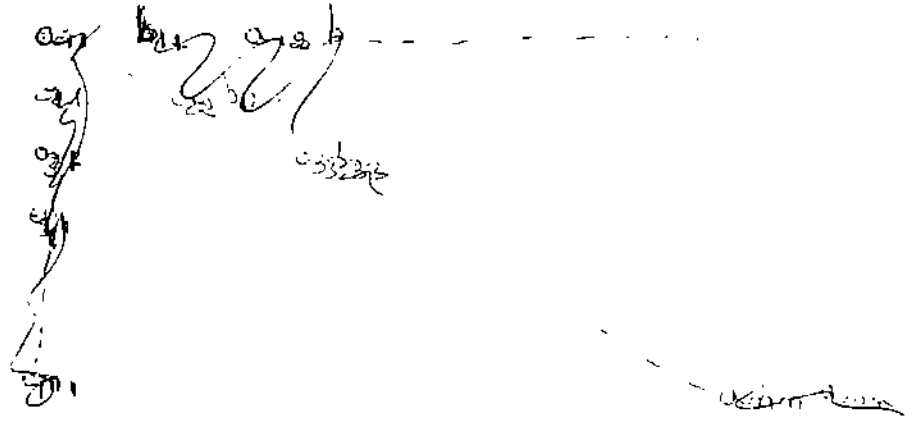
c) The reflexive closure of R is a new relation  $R' = R \cup \Delta$  where  $\Delta = \{(a, a) \mid a \in A\}$

$$R' = \{(1,1), (1,3), (1,4), (2,2), (2,3), (2,4), (3,1), (3,3), (3,4), (4,4)\}$$

d) The symmetric closure of  $R$  is a new relation  
 $R' = R \cup R^{-1}$  where  $R^{-1} = \{ (b, a) / (a, b) \in R \}$

$R' = \{ (1, 3), (3, 1), (1, 4), (4, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 1), (3, 4), (4, 3) \}$

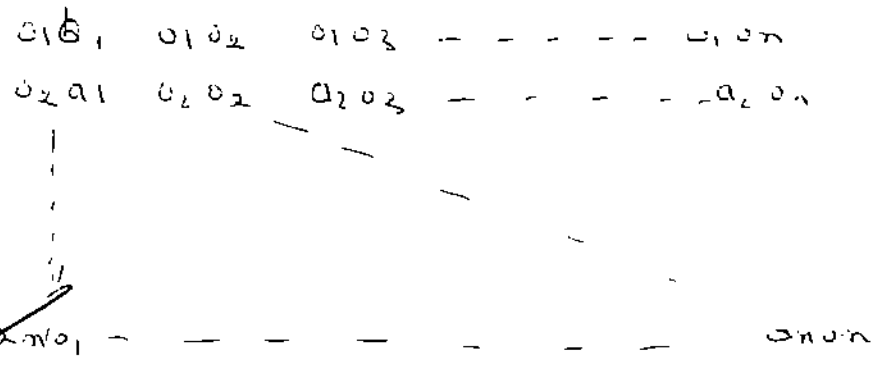
7) lets represent the  $A \times A$  as a matrix form.



**5**

Answer =  $2 \frac{n(n-1)}{2}$

But your ratio is not correct



The relation is a subset of  $A \times A$ .

The symmetric relation  $R' = R \cup R^{-1} \Rightarrow$  elements above the diagonal

are  $(m-1) + (m-2) + \dots + 2 + 1$  and with including the diagonal  $\frac{n(n+1)}{2}$  element.

The number of elements that are reflexive of the diagonal is equal to  $D = m$   $A = \{ (a, a) / a \in A \}$

The number of ~~subset~~ relations that are reflexive and symmetric ~~subset~~  
 $\frac{n(n-1)}{2}$   $\hookrightarrow 2 \frac{n(n-1)}{2}$

Mat 211 – Discrete Mathematics  
Exam # 2

Solve the following problems

- 1) a – The sequence  $f(n)$  is defined recursively as follows  
 $f(1) = 1, f(n) = 2f(n-1)$  for  $n \geq 2$ . Find a formula for  $f(n)$  **10**  
b – Give a recursive definition of the sequence  $\{b_n\}, n=1,2,\dots$   
if  $b_n = 4n - 2$ . **10**  
(20 pts)
- 2) a -A particular brand of shirts comes in 12 colors, has a male version  
and a female version, and comes in three sizes for each sex.  
How many types of this shirt are made? **10**  
b -A multiple choice test contains 10 questions. There are 4 possible answers for  
each question. How many ways can a student answer the questions on the test  
if every question is answered?  
(20 pts)
- 3) What is the minimum number of students, each of whom comes from one of 50  
high schools, enrolled in a university to guarantee that there are at least 10 who  
come from the same high school? **4 < 1**  
(10 pts)
- 4) Suppose that a department contains 3 men and 4 women. How many ways are  
there to form a committee with 4 members if it must have the same number of  
men and women? **18 committee**  
(10 pts)
- 5) What is the coefficient of  $x^2 y^8$  in the expansion of  $(2x + 3y)^{10}$ ? **✓**  
(10 pts)
- 6) Let  $R = \{ (1,3), (1,4), (2,3), (2,4), (3,1), (3,4) \}$  be a relation on the set  $\{1,2,3,4\}$   
a) Find a matrix representation of R  
b) Represent R by a directed graph  
c) Find the reflexive closure of R  
d) Find the symmetric closure of R **✓**  
(20 pts)
- 7) How many relations are there on a set with n elements that are reflexive and  
symmetric?  
(10 pts)